A workflow to model anisotropy in a vertical transverse isotropic medium

M. Reza Saberi\textsuperscript{*} and Jimmy Ting\textsuperscript{1} review anisotropy in vertically isotropic media and use existing theory to model changes in the elastic stiffness tensor based on conventional well logs.

Velocities in anisotropic media, which is known as the directional dependency of velocities, is becoming increasingly important in subsurface imaging and characterization. Most elasticity theories consider an isotropic medium to describe the phenomena in the field of reservoir geophysics. This assumption is challenged by the reality of the subsurface which is subject to a complex geological history such as tectonic movements and changes in the differential stress that can typically introduce fractures. In some cases, these factors can make the subsurface highly anisotropic. In general, four classes of anisotropy can be defined, ranging between the two extremes of a completely isotropic medium (with two elastic constants) and a completely anisotropic medium (with 21 elastic constants). The four classes refer to specific conditions where we can reduce the number of elements of the elastic stiffness tensor. These are known as Cubic (with three independent elastic constants), Transverse Isotropic or TI (five independent elastic constants), Orthorhombic (nine independent elastic constants) and Monoclinic (13 independent elastic constants). The purpose of this paper is to review anisotropy in vertically isotropic media and use existing theory to model changes in the elastic stiffness tensor based on conventional well logs. Furthermore, this elastic stiffness tensor can be used to calculate the Thomsen parameters or even to model anisotropic velocities directly.

Review of anisotropy and elastic stiffness

Anisotropy as an extension to isotropic approaches is usually dealt with using Thomsen (1986) parameters as approximations. Thomsen (1986) suggested three parameters to correct for anisotropy effects in weakly anisotropic media. These parameters, $\varepsilon$, $\delta$ and $\gamma$, are regularly used in all reservoir geophysics disciplines to address anisotropy effects. However, determining these three parameters is not straightforward and requires information such as laboratory data or well logs acquired in boreholes in different directions with respect to the symmetry axis of the anisotropy. The purpose of this paper is to review anisotropy in vertically isotropic media and use existing theory to model changes in the elastic stiffness tensor based on conventional well logs.

Figure 1 Stiffness tensor and velocities for an isotropic medium.

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Anisotropy in TI media and Thomsen parameters

TI media are defined as materials that show isotropic behavior in one plane and anisotropic behavior in the direction perpendicular to the isotropic plane (Figure 2). The direction of the anisotropy is normally referred to as the symmetry axis. In such conditions, the number of independent constants in the $C_v$ matrix increases to five, and isotropic velocities will change accordingly:

$$V_{QP} = \frac{c_{10} \sin^2 \theta + c_{33} \cos^2 \theta + c_{44} + \sqrt{M}}{2\rho}$$

$$V_{QSV} = \frac{c_{10} \sin^2 \theta + c_{33} \cos^2 \theta - c_{44} - \sqrt{M}}{2\rho}$$

$$V_{SH} = \frac{c_{16} \sin^2 \theta + c_{44} \cos^3 \theta}{\rho}$$

$$M = [(c_{33} - c_{44}) \sin^2 \theta - (c_{33} - c_{44}) \cos^2 \theta]^2 + (c_{11} + c_{44}) \sin^2 2\theta$$

Here, $V_{QP}$ and $V_{QSV}$ are the Quasi-longitudinal mode and Quasi-shear mode velocities, while $V_{SH}$ is the horizontal shear velocity. $\theta$ is the angle between the wave vector and the symmetry axis of the TI medium. Thomsen (1986) approximated these velocities in a TI medium for a weakly elastic anisotropic scenario as follows:

$$V_{QP} \approx V_p(0)[1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta]$$

$$V_{QSV} \approx V_s(0)[1 + \frac{V_p(0)}{V_s(0)} (\epsilon - \delta) \sin^2 \theta \cos^2 \theta]$$

$$V_{SH} \approx V_s(0)(1 + \gamma \sin^2 \theta)$$

Here, $\epsilon$, $\delta$ and $\gamma$ which are known as Thomsen anisotropic parameters are defined as:

$$\epsilon = \frac{c_{11} - c_{33}}{2c_{33}}$$

$$\delta = \frac{(c_{11} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}$$

$$\gamma = \frac{c_{16} - c_{44}}{2c_{44}}$$

and $V_p(0)$ and $V_s(0)$ are the velocities along the symmetry axis with the following definitions:

$$V_p(0) = \frac{c_{33}}{\rho}$$

$$V_s(0) = \frac{c_{44}}{\rho}$$

These two velocities, which represent the slowest velocities in the TI medium, can also be calculated using equation (2) with $\theta=0$. In other words, Thomsen’s equation calculates velocities in the symmetry direction and then adds the anisotropy effects into these velocities using the three Thomsen parameters. This means that the five independent elastic parameters which have no physical meaning are now translated into the two isotropic velocities and three parameters ($\epsilon$, $\delta$ and $\gamma$) with a physical meaning. $\epsilon$ is also referred to as P-wave anisotropy and represents the fractional difference between the P-wave velocities in the horizontal and vertical directions. $\delta$ can be related to both near-vertical P-wave velocity and angular SV-wave velocity variations. $\gamma$ has the same role as $\epsilon$ but for S-wave velocity by representing the fractional difference between the SH-wave velocities in the horizontal and vertical directions. Therefore, these three parameters are used to describe a TI medium, and a measure of them is essential in any anisotropic modelling workflow. This is the main motivation for many studies after Thomsen (1986). These studies work with the three Thomsen parameters directly or with a combination of them (e.g. Alkhalifah and Tsvankin, 1995) to model wave propagation in an anisotropic medium. However, the Thomsen approximation is only valid for weak anisotropy and calculation of $\epsilon$, $\delta$ and $\gamma$ is not an easy task. In the following sections, the anisotropy problem in TI media will be reviewed by looking at anisotropy effects on the stiffness matrix $C_v$ using the Backus model. Then, a workflow which only needs conventional well logs will be proposed to model anisotropy in such a medium. This approach requires good knowledge of the source of the anisotropy, and can be
coupled easily with other anisotropic workflows to enhance accuracy of anisotropic modelling.

**Backus model and stiffness tensor in a VTI medium**

Postma (1955) showed that in heterogeneous media anisotropy is a scale-dependent property where a two-layer layered medium can behave as an anisotropic medium if each of the two contributing layers is isotropic at a finer scale than the wavelength of the seismic waves. Backus (1962) extended Postma’s work into general media with three or more layers. Backus (1962) showed that in the long wavelength limit, a stratified medium composed of isotropic layers will behave like a TI medium. This theory allows a set of isotropic layers to be replaced by a single anisotropic layer or a single anisotropic medium to be decomposed into a set of isotropic layers. This implies that anisotropy is a frequency-(scale) dependent phenomenon, and fine-scale isotropic layering (higher frequency) can express itself as anisotropic on a larger scale (lower frequencies). Seismic velocity is a scale-dependent property (i.e. laboratory, well logs or seismic measurements) in which inferred velocity is slower when the wavelength of the propagating wave is longer than the geological heterogeneity, and this wavelength differs for different measurements. The wavelength of a propagating wave ranges approximately from millimetres for laboratory measurements to centimetres/metres for well logs and many metres for seismic measurements. Therefore, normally it is advised to replace a stratified medium of fine layering in the well log scale with a homogeneous, transversely isotropic material in seismic scale. As a matter of fact, anisotropy can be defined on different scales from large scale (e.g. medium layering or fractures with scales of many metres) down to fine scale (e.g. grain alignments or cracks with scales of millimeters or even less), and it is the observation scale that determines if that medium behaves as an isotropic or anisotropic medium. For normal incidence seismic wave propagation, when these anisotropy features (such as stratified media) are on a scale much finer than the wavelength of seismic waves, the waves will average their elastic properties, and the medium will behave as a homogeneous effective medium. In this regard, the Backus average is normally considered as the low frequency limit while ray theory defines the high frequency limit of the medium velocity (Mavko et al., 2009).

Backus (1962) derived the effective elastic constants for a stratified medium composed of TI layers in the long wavelength limit. This method replaces the stratified medium with an equivalent TI medium, and the component fine layers could be either isotropic or anisotropic with spatially periodic or non-periodic pattern. In the case of TI fine layers, the general elastic stiffness constants for the equivalent TI medium can be written as below using the Backus model:

\[
\begin{align*}
C_{11} &= \left( c_{11} - c_{13}^2 c_{33}^{-1} \right) + \left( c_{33}^{-1} \right)^{-1} \left( c_{13} c_{33}^{-1} \right)^2 \\
C_{12} &= \left( c_{12} - c_{13} c_{35}^{-1} \right) + \left( c_{35}^{-1} \right)^{-1} \left( c_{13} c_{35}^{-1} \right)^2 \\
C_{13} &= \left( c_{33}^{-1} \right)^{-1} \left( c_{13} c_{33}^{-1} \right) \\
C_{33} &= \left( c_{33}^{-1} \right)^{-1} \\
C_{44} &= \left( c_{44} \right)^{-1} \\
C_{55} &= \left( c_{55} \right)^{-1} \\
C_{66} &= \left( c_{66} \right)^{-1}
\end{align*}
\]

The brackets indicate averages of the enclosed properties weighted by their volumetric proportions, \( \bar{c} \) and \( \bar{c}_e \) refer to the constants of an elastic TI equivalent medium and to fine layers respectively. If the individual fine layers are isotropic, the equivalent medium is still a TI medium. The elastic constants of such a TI medium can be calculated using equation...
This means that all anisotropic minerals, such as clay, or any other anisotropic factors, such as fractures and cracks, should be considered separately in the first layer, while the rest of the medium with isotropic behaviour constitutes the second layer. Based on the Backus model, the equivalent effective medium resulting from these two layers (isotropic or anisotropic) behaves as a TI medium (Saberi, 2016). This means that splitting of a given medium into two separate layers secures a TI behaviour for the equivalent effective medium. However, the assumption of the combination of isotropic and anisotropic layers enables us to manipulate the degree of the anisotropy for the equivalent medium in a way that the lowest degree of anisotropy will occur when both layers are isotropic (without any rock physics modelling) and highest anisotropy will occur when rock physics modelling is performed to include other anisotropic factors in the anisotropic layer.

The proposed workflow for modeling an anisotropic medium is shown in Figure 3. This figure explains the workflow in four steps. It starts by dividing the medium into two nominal layers (step 1) and continues with rock physics modelling of the first layer (assumed to be the TI medium) along its symmetry axis by combining all the initial components and elements and using a suitable rock physics model (step 2). The output of this step (step 2) is an effective medium that averages all elastic properties and fractions of the initial components within the anisotropic layer. Indexes of 1 and 2 refer to the layer number.

Equation (7) assumes that the layers (isotropic and anisotropic) are on a much finer scale than the seismic wavelength. This means that the waves will average the physical properties of the fine layers, so that the material becomes a homogeneous effective medium with these velocities for a plane wave propagation normal to the layering.

**Workflow to model elastic constants in a TI medium**

TI media can be considered as a periodic stack of two layers, either isotropic or anisotropic or a combination. Here we assume a model consisting of two nominal layers: the first layer containing all anisotropic factors and components while the second layer will represent the rest of the medium without any anisotropic behaviour. Note that the splitting of the medium into two layers must be done considering the physical properties of the medium. In general, the choice of the separate layers will be based on auxiliary knowledge or measurements. This means that all anisotropic minerals, such as clay, or any other anisotropic factors, such as fractures and cracks, should be considered separately in the first layer, while the rest of the medium with isotropic behaviour constitutes the second layer. Based on the Backus model, the equivalent effective medium resulting from these two layers (isotropic or anisotropic) behaves as a TI medium (Saberi, 2016). This means that splitting of a given medium into two separate layers secures a TI behaviour for the equivalent effective medium. However, the assumption of the combination of isotropic and anisotropic layers enables us to manipulate the degree of the anisotropy for the equivalent medium in a way that the lowest degree of anisotropy will occur when both layers are isotropic (without any rock physics modelling) and highest anisotropy will occur when rock physics modelling is performed to include other anisotropic factors in the anisotropic layer.

Figure 3: A simple flowchart showing how the proposed workflow works to model anisotropic behaviour using conventional well logs. If both layers are considered as isotropic then the simplest case of VTI will be modelled and is mainly related to the fine layering in the medium (modified after Saberi (2016)).
can be used to include such information into the first layer. If the source of anisotropy is related to extrinsic factors such as cracks and fractures, then rock physics models such as Hudson (1980) can be used to model the first layer as a fractured medium. Here we should assume that all cracks with a preferential direction are located within the first anisotropic layer while the non-fractured part will make the second isotropic layer. If the anisotropy source is considered to be the repetition of two isotropic layers where fine layering (scale effects as discussed earlier) between two isotropic media causes anisotropic behaviour of the effective medium, then we can assume that the first isotropic layer elastic parameters are simply characterized by $\lambda$, $\mu$, and $\rho$. The selection of the values of these elastic parameters depends on the lithology of the isotropic layer.

Anisotropic minerals such as clay can have quite a wide range of elastic properties due to the mineral type or depth-induced changes related to diagenesis. Even the Hudson (1980) model requires some detailed information on crack and fracture characteristics. Therefore, external knowledge of the cause of the anisotropy is crucial in the modelling process, since the degree of anisotropy of the medium will be inferred based on the results of this step (step 2). One possible approach to validate this information is to couple this workflow with other measurements such as core ultrasonic measurement for different angles. This means that, for instance, input mineral elastic properties or crack properties will be updated based on the core measurement data at different incident angles. In this regard, boundary models such as the Hashin-Shtrikman (1963) boundary model or Voigt (1890) and Reuss (1929) models can be used to define the boundaries for anisotropic changes in a given medium.

The next step (step 3) is to calculate elastic properties of the second layer (isotropic medium) using first layer information along with measured elastic logs and equation 7. Equation 7 assumes a vertical well and horizontally stratified layer. Here, we simply subtract the modelled elastic properties from the measured elastic properties to calculate their residual properties and consider them as the isotropic medium elastic properties for the second layer. Note that here all of the calculations are done along the symmetry axis and the anisotropic layer will have one set of Lamé parameters along this axis ($\lambda_{\text{symmetry}}$ and $\mu_{\text{symmetry}}$). This step downscales the measured elastic logs into two equivalent layers in such a way that their summation is equivalent to the measured one along the symmetry axis.

Finally, step four uses equation (6) to sum up the two downscaled layers to rebuild the measured elastic logs. The upscaled velocity logs along the symmetry axis will be identical to the measured velocity logs as the same logs were used in generating the second layer elastic properties. However, deviating from the symmetry axis will result in velocity changes based on the modelled elastic stiffness tensor components. These five elastic constants will change in...
such a way that the modelled logs will always be equivalent to the measured log along the symmetry axis (θ=0). But the (anisotropic) information incorporated in the second step will affect modelled velocities when the incident angle of wavefront is different from zero (θ≠0). Finally, equation (2) can be used to model velocities based on the modelled elastic constants at different incident angles (θ).

Case study and discussion
The presented rock physics workflow proposes a four-step approach to model elastic constants in a TI medium. These steps reconstruct measured velocities \( V_p \) and \( V_s \) by downscaling and upscaling the TI medium using the Backus model along the symmetry axis.

The simplest scenario occurs if the source of anisotropy (TI medium) can be considered to be fine layering (Figure 2). This scenario assumes repetition of two isotropic layers for the equivalent TI medium. This seldom occurs in reality as there are additional factors such as fractures that will cause a layer to behave as an anisotropic medium. In any case, rock physics modelling of layer one, the anisotropic layer, is crucial. The main requirement for this step is to model the anisotropic velocity in step 2 along the symmetry axis, which further infers that the well logs should have been acquired along this axis. Having the anisotropic velocities along the symmetry axis enables us to calculate one set of Lamé parameters along this axis. This set of Lamé parameters represents the anisotropic medium elastic properties for \( \theta = 0 \).

The more general case for this workflow is the scenario where anisotropy is expected owing to repetition of more than two types of isotropic layers. This knowledge could come from well logs, geology information or even laboratory

![Figure 5 Modelled velocities for different incident angles: (a) \( \theta = 0^\circ \), (b) \( \theta = 45^\circ \), and (c) \( \theta = 90^\circ \) in a VTI medium penetrated by a vertical well. The black and red curves in the first two tracks from left are the measured and modelled logs for \( V_p \) and \( V_s \). The green curve shown in the second track for (b) is the modelled \( V_{QSV} \) which is equal to \( V_{SH} \) for \( \theta = 0^\circ \) and \( \theta = 90^\circ \) (a and c). Core measurements are shown as points (three core samples at three different depths) with different colours. Blue is the core velocity measurements for \( \theta = 0^\circ \), green for \( \theta = 45^\circ \) and black for \( \theta = 90^\circ \). The lithology fraction track shows the results of the first step in the workflow (Figure 3) where the lithology is divided into anisotropic (black) and isotropic (red) media. The white interval is the evaluation interval, and the anisotropy parameters used in this workflow are calibrated for this interval.](image-url)
measurements and observations. In this more general case, we can cascade the proposed process. First, a TI medium with two isotropic layers should be built using the described workflow. Then, the result of this step should be used as the input to the second step in the proposed workflow which models the first anisotropic layer elastic properties (TI medium). Step three follows to model the second layer, the isotropic layer. This process can be repeated until all of the isotropic layers are included in the TI medium. In this case, the final equivalent medium from the first loop will provide the effective medium required in step two and so on. If another reason for the first layer anisotropy (TI medium) is expected then an appropriate rock physics model is required to include such anisotropy into these layer elastic properties. The accuracy of these modelled anisotropic elastic properties needs to be confirmed using additional data. This extra information for TI media can come from other disciplines such as ultrasonic measurements on cores or even azimuthal seismic inversion. Ultrasonic core measurements where velocities at different incident wave angle are provided are a good candidate in this regard. This means that the modelling parameters in the second step should be updated in accordance with the best fit of the final equivalent medium velocities and velocities coming from ultrasonic measurements at different wave incident angles. However, without such information it is only possible to define a range for changes in the TI medium elastic constants. This range can represent scenarios from low to high anisotropy, where the actual anisotropy could be located in the range between these extremes.

The described workflow was tested on a well drilled into the Duvernay shale formation, which can be considered as a VTI medium, in the Western Canadian sedimentary basin. This well has the following conventional well logs: P-sonic, S-sonic, gamma ray, resistivity, density and neutron porosity along with three core samples at different depths within the shale interval (area of interest). For these three samples, all the data needed for this study are measured: porosity, mineralogy and velocities. The P-wave and S-wave sonics are available at 0°, 45° and 90° in order to find the best set of these properties that match all three velocities for all samples based on the given 0°, 45° and 90° in order to find the best set of these properties that match all three velocities for all samples based on the given 0° (equation 2). Figures 5a, b and c show the modelling results for the tuned parameters following the given workflow for three incidence wave angles (0, 45 and 90 degrees). These are the angles for which core velocity measurements are available. A good match between the measured and modelled data at different wave incident angles (0°) can be observed. Furthermore, these three velocities (related to different angles) can be used to calculate the log for the Thomsen parameters within the given interval.

**Conclusions**

This paper reviews anisotropy in TI media and discusses a rock physics modelling workflow to model the elastic stiffness tensor in such media. This is a fast approach for estimating anisotropy parameters using only conventional well logs. This workflow couples different rock physics models with the Backus model in an attempt to extract elastic constants.
by downscaling and upscaling elastic properties. The normal incidence Backus model is used to downscale logs into two nominal layers used to describe the isotropic and anisotropic medium properties along their symmetry axis, and then the full Backus model is applied on the same layers to upscale them to the initial medium. This approach can be coupled easily with any other workflows, such as VTI anisotropic inversion, to model VTI mediums. The introduced workflow was tested on a well with conventional logs and core measurement samples, and a set of anisotropic parameters were derived to match measured core sample velocities at different incident angles with respect to the symmetry axis. In addition, the rock physics modelling workflow applied here is also able to distinguish between anisotropy caused by pyrite particle versus clay particles.

References